



Division of
Academic Enhancement
UNIVERSITY OF GEORGIA

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Department of Mathematics
Franklin College of Arts and Sciences
UNIVERSITY OF GEORGIA

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MATH 1113 Exam 1 Review

Spring 2018

Topics Covered

Section 1.1: Rectangular Coordinate System

Section 1.2: Circles

Section 1.3: Functions and Relations

Section 1.4: Linear Equations in Two Variables and Linear Functions

Section 1.5: Applications of Linear Equations and Modeling

Section 1.6: Transformations and Graphs/Compositions

Section 1.7: Analyzing Graphs of Functions and Piecewise Functions

Section 1.8: Algebra of Functions and Composition of Functions

Section 2.1: Quadratic Functions and Applications

Section 1.1: Rectangular Coordinate System

The xy -Plane

The xy -plane consists of a horizontal axis (the x -axis) and a vertical axis (the y -axis).

The origin is located at the intersection of the two axes.

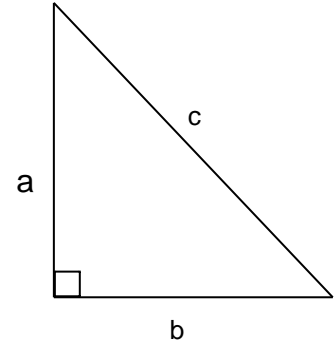
Locations on the xy -plane are denoted by ordered pairs (x, y)

The Pythagorean Theorem

Given a right triangle, the lengths of the sides are related by the following equation:

$$a^2 + b^2 = c^2$$

Where a and b are the sides (legs) that form the right angle and c is the hypotenuse of the triangle.



The Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) is derived from the Pythagorean Theorem and found by using the following equation:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The Midpoint Formula

The midpoint of a line segment joining points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Examples

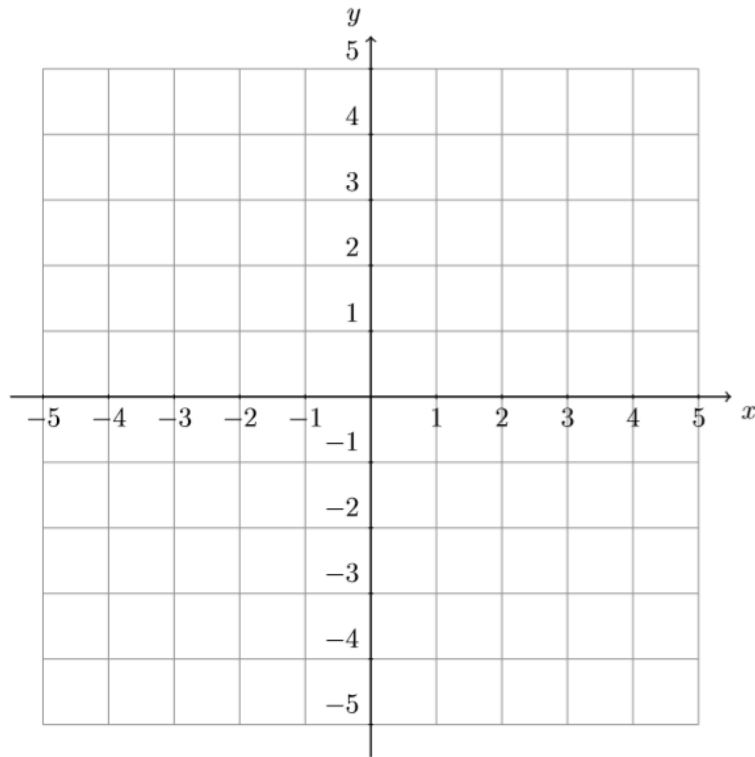
1. Answer the following

(a) Calculate the distance from $P(-1, -1)$ to $Q(2, 3)$.

(b) Find a point R such that R is the midpoint of PQ .

(c) Plots points P and Q on the coordinate plane below.

(d) On the coordinate plane, sketch a picture with labels that show how the components of the distance formula appear geometrically.



2. Find all points on the x -axis that are a distance 5 units away from $P(3,4)$.

Section 1.2: Circles

The Equation of a Circle in Standard Form

The equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Examples

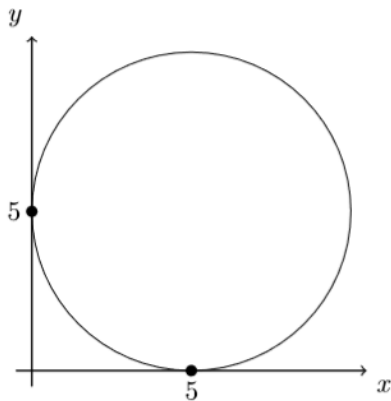
3. Answer the following about the circle $x^2 + 2y = 6x - y^2 - 7$

(a) What is the center and radius of the circle?

(b) Find the intercepts of the circle (if they exist).

(c) Find the points where the line $y = -1$ intersects the circle.

4. Determine the equation of the circle shown below that is tangent to the y -axis at $(0,5)$ and tangent to the x -axis at $(5,0)$.



Section 1.3: Functions and Relations

Functions

A function from A to B is a rule of correspondence that assigns each element of A to exactly ONE element of B.

Vertical line test

A graph in the xy -plane represents a function $y = f(x)$ provided that any vertical line intersects the graph in at most one point.

Domain (the inputs)

The domain of a function $f(x)$ represents the set of all x values allowed to go into $f(x)$.

Finding the domain **graphically**: Read the function from left to right

Finding the range **algebraically**: Check for one of the following three 'road blocks'.

1. Exclude any values of x that would make a denominator equal to zero
2. Exclude any values of x that would make expressions under even roots negative.
3. Exclude any values of x that would make expressions inside log functions non-positive.

Range (the outputs)

The range of function $f(x)$ represents the set of all y values allowed to come out of $f(x)$.

Finding the range graphically: Read the function from bottom to top

Examples

5. Determine the domain of the following functions:

(a) $f(x) = x^3 - 2x^2 + x + 13$

(b) $f(x) = \frac{-4}{x^2-1}$

(c) $f(x) = \sqrt{2x-5}$

(d) $f(x) = \frac{1}{\sqrt{x-7}}$

Section 1.4: Linear Equations in Two Variables and Linear Functions

The Slope-Intercept Formula

The equation of a line with slope m and y -intercept b is:

$$y = mx + b$$

The Point-Slope Formula

The equation of a line with slope m passing through the point (x_1, y_1) is:

$$y - y_1 = m(x - x_1)$$

Equations of Horizontal and Vertical Lines

The equation of a horizontal line through the point (a, b) is $y = b$. (All run, no rise)

The equation of a vertical line through the point (a, b) is $x = a$. (All rise, no run)

Parallel and Perpendicular lines

Two non-vertical lines with slopes m_1 and m_2 are parallel if and only if

$$m_1 = m_2$$

Two non-vertical lines with slopes m_1 and m_2 are perpendicular if and only if

$$m_1 = -\frac{1}{m_2}$$

Examples

6. Determine the equations satisfying the conditions stated.

(a) A straight line passing through points $(-2, 1)$ and $(4, 6)$.

(b) A straight line having slope $m = \frac{3}{11}$ passing through point $(-3, 4)$.

(c) A straight line passing through $(-2, 1)$ and perpendicular to the line from (b).

Section 1.5: Applications of Linear Equations and Modeling

Examples

7. The value of a newly purchased boat is a linear function of time. The boat was purchased for 15,000 dollars. If the value of the boat decreases to 75 percent of its purchase price in 3 years, determine the value of the boat after 7 years.

8. Kim really loves donuts. She is also concerned about her health so she wants to track how many calories a week she consumes due to her donut habit. If she lets x be the number of donuts she eats in a week, y be the number of calories consumed per week from donuts and assuming a linear relationship between x and y , answer the following.

(a) Should the slope of the line be positive, negative or zero? Why do you think so?

(b) Should the y-intercept be positive, negative or zero? Why do you think so?

Section 1.6: Transformations and Graphs

Translations and Reflections of Functions

$$y = f(x) \pm c$$

Translate c units vertically upward/down

$$y = f(x \pm c)$$

Translate c units to the left/right

$$y = -f(x)$$

Reflect about the x -axis

$$y = f(-x)$$

Reflect about the y -axis

$$y = cf(x)$$

Stretches or compresses vertically by a factor of c

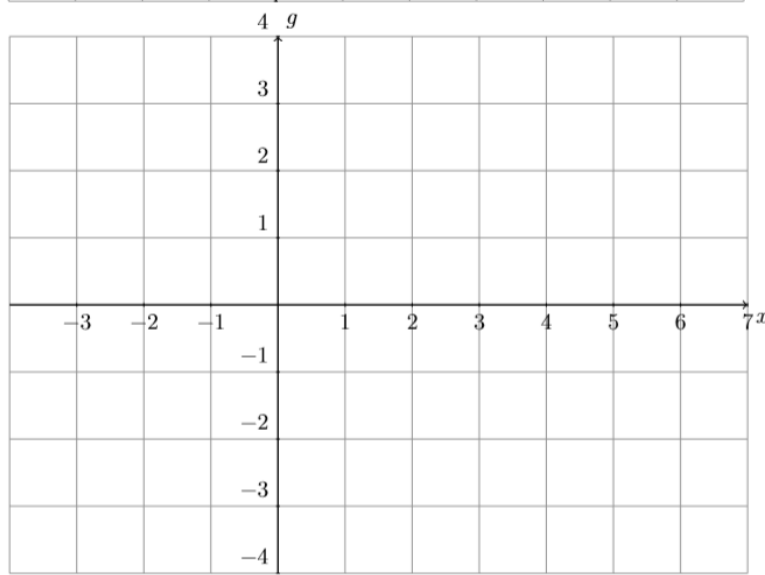
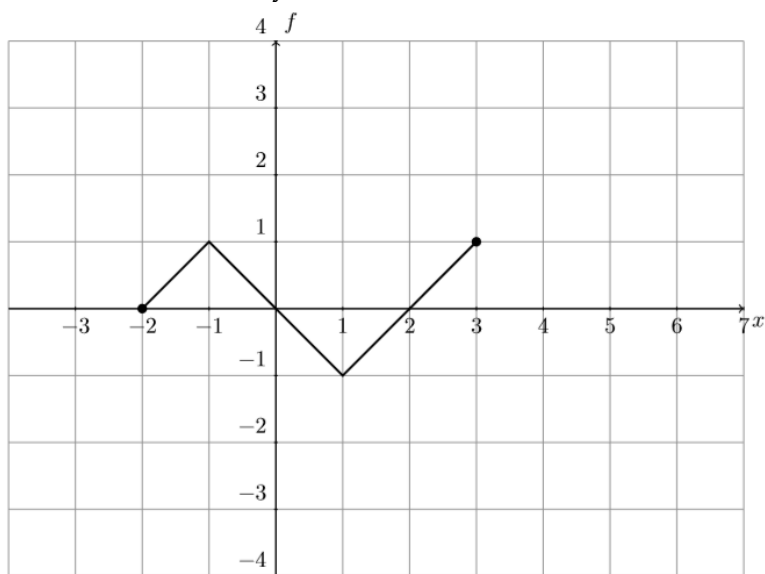
$$y = f(cx)$$

Stretches or compresses horizontally by a factor of c

NOTE: When making multiple movements, do all reflections \rightarrow stretches \rightarrow shifts

Examples

9. Using the graph of $f(x)$, draw the graph of $g(x) = 2f(x + 1) - 1$ on the blank graph provided. List all the transformations you made.



10. Write a new function $g(x)$ based on the given function $f(x)$ and the transformations in the given order.

$$f(x) = \sqrt{x}$$

1. Shift 5 units to the left
2. Vertically stretch by a factor of 2
3. Reflect across the x -axis
4. Shift downward 3 units

(a) Write the new equation for $g(x)$.

(b) Sketch the graph of $g(x)$



Section 1.7: Analyzing Graphs of Functions and Piecewise Functions

Symmetry of Odd and Even Functions

If $f(-x) = f(x)$, then $f(x)$ is an even function and symmetric to the y -axis. (Ex: $y = x^2$)

If $f(-x) = -f(x)$, then $f(x)$ is an odd function and symmetric to the origin. (Ex: $y = x^3$)

Piecewise Functions

Piecewise functions are functions that are made up of pieces of other functions.

Plugging in values of x :

1. Look at the column on right side and find the interval that contains the desired value of x .
2. Use the corresponding piece of the function in the left column to evaluate.

Sketching a piecewise function

1. Draw the function from the left column using a dotted line
2. Use the corresponding interval in the right column to fill in the allowed values with a solid line
3. Remove the remaining unused part of the function
4. Repeat for each piece of the function.

Examples

11. Determine whether each of the following functions are even, odd, or neither. Justify your answers algebraically.

(a) $f(x) = x|x|$

(b) $g(x) = \sqrt{x} + 2$

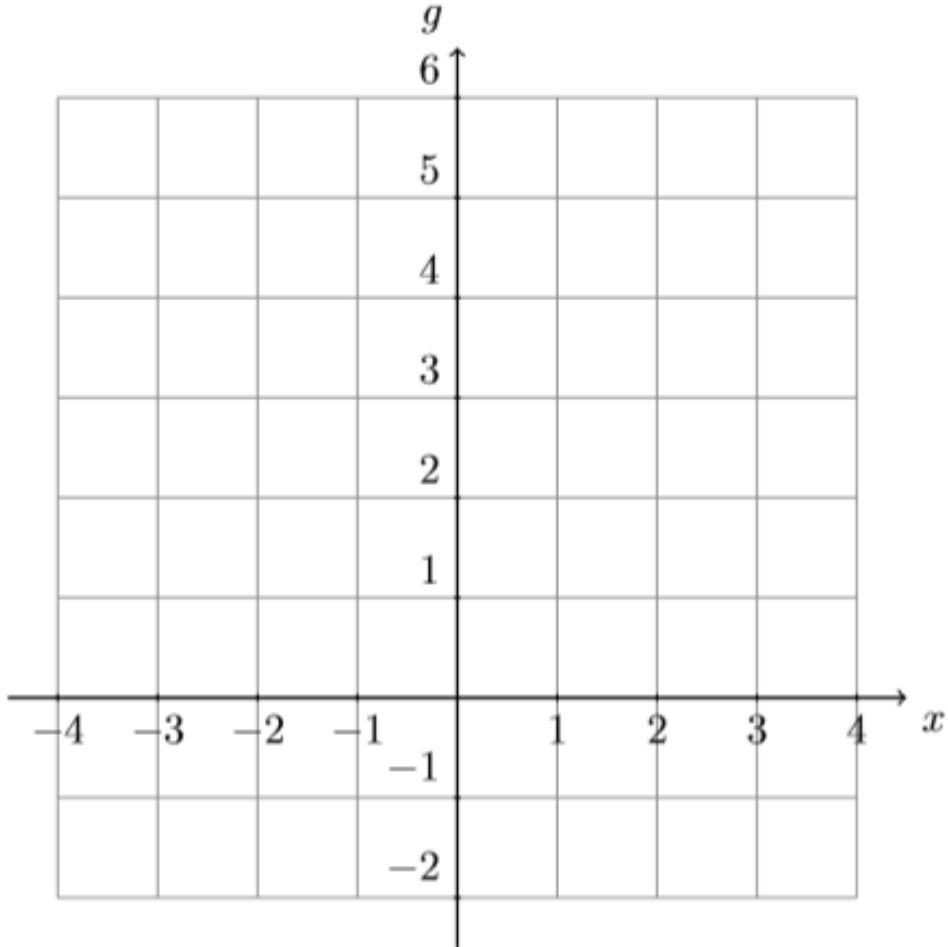
(c) $h(x) = 4x^4 - x^2$

(d) $k(x) = \frac{x}{x^2+1}$

12. Use the piecewise defined function, g , to answer the questions below.

$$g(x) = \begin{cases} 3x + 9, & -3 < x \leq -1 \\ x^2 - 1, & -1 < x < 2 \\ x - 4, & 3 \leq x \leq 4 \end{cases}$$

(a) Sketch a graph of the function on the graph below.



(b) Determine the domain of the function.

(c) Determine the range of the function.

Section 1.8: Algebra of Functions and Composition of Functions

Algebraic Properties of Functions

Let $f(x)$ and $g(x)$ be functions of x .

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$

Composition of Functions

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

To evaluate a composite function, work from the inside out!

The Difference Quotient

$$m = \frac{f(x+h) - f(x)}{h}$$

Examples

13. Given the functions

$$m(x) = x - 4, \quad n(x) = \frac{1}{x - 3}, \quad p(x) = \sqrt{x - 5}$$

(a) Find $(mn)(x)$ and state the domain in interval notation.

(b) Find $\left(\frac{m}{p}\right)(x)$ and state the domain in interval notation.

14. Use the two functions below to answer the following.

$$f(x) = x^2 - 1, \quad g(x) = \sqrt{2x + 1}$$

(a) Determine the value of $g(f(4) + 1)$.

(b) Write a formula which defines $f(2g(x - 1))$

15. Given $f(x) = -2x^2 + 7x - 3$, find

(a) $f(-1)$

(b) The difference quotient

(c) The average rate of change of f on the interval $[1, 3]$.

Section 2.1: Quadratic Functions and Applications

Linear Functions vs. Quadratic Functions

A linear function is of the form $f(x) = ax + b$ (polynomial of degree 1) while a quadratic function is of the form $f(x) = ax^2 + bx + c$ (polynomial of degree 2)

By completing the square, the equation of the parabola $y = ax^2 + bx + c$ can always be rewritten in the form $y = a(x-h)^2 + k$ where (h, k) is the vertex of the parabola.

Examples

16. A kayak rental business operates along a river in Georgia. Suppose the business has determined that for any number of kayak, x , it rents, the price function is $p(x) = 60 - 12x$.

(a) Determine the equation for the total revenue $R(x)$ for the rental of x kayaks. (Hint: the revenue is found by multiplying the price of kayak rentals by the number of kayaks rented.)

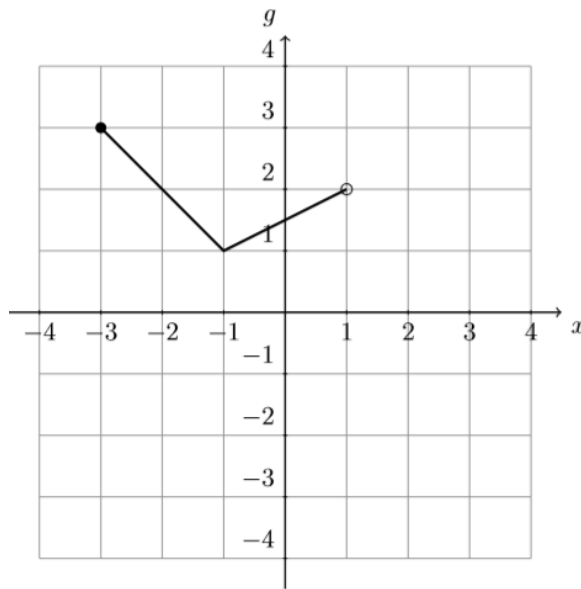
(b) What is the maximum revenue the owners can make in this kayak business?

(c) What price should they charge to make the maximum revenue?

17. A diverter at the end of a gutter spout is meant to direct water away from a house. The homeowner makes the diverter from a rectangular piece of aluminum that is 20 inches long and 12 inches wide. She makes two folds both parallel to the 20 inch side. Each fold is a distance x away from the edge. What is the maximum volume of water that can be carried through the diverter?

Practice Problems

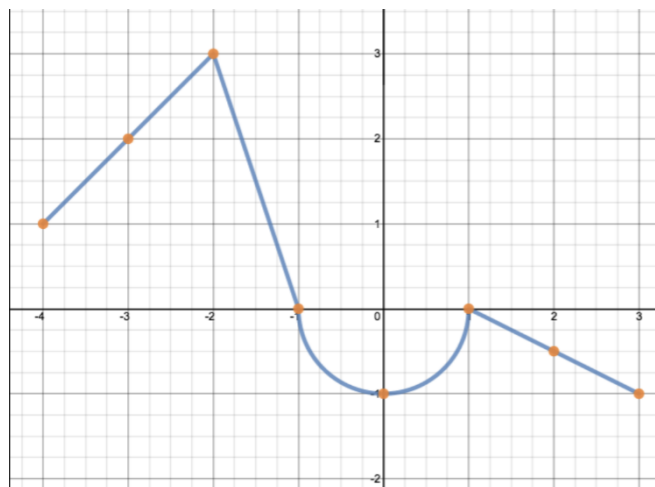
1. Given $P(-2,3)$ and $Q(8,-5)$, find
 - (a) The distance of PQ .
 - (b) The midpoint of PQ .
 - (c) A point R such that Q is the midpoint of PR .
2. Find an equation of a circle that has a center at $C(-2,-8)$ and is tangent to the line $x = 2$.
3. Find the center and radius of the circle $3x^2 + 24y - 18x + 3y^2 - 33 = 0$.
4. Find the domain of $f(x) = (x + 7)^{1/2}$, $g(x) = (x - 3)^{-1/2}$
5. Find the slope and equation of the line that passes through point $(4,7)$ and is perpendicular to the line $x + 6y + 4 = 0$.
6. Binky's snow cone stand typically sells 200 snow cones when they price them at \$4.50. However, for every \$0.50 reduction in price, they will sell 40 additional snow cones.
 - (a) Build a linear model for Binky's sales as a function of price.
 - (b) How many snow cones will they sell if they price them at \$6.00?
 - (c) Binky's goal is to sell 420 snow cones in a single day. What price should they price them at?
 - (d) What is the domain and range of this linear model?
7. Point $P(1,2)$ lies on the graph of $f(x)$. The domain of $f(x)$ is $[0,10]$ and the range is $[-3,8]$.
 - (a) Find the location of P on $y = -2f(4x) - 6$.
 - (b) Find the domain and range for $y = -2f(4x) - 6$.
8. The graph of $g(x)$ is given in the plot below. Use the graph to answer the questions below.
 - (a) Determine the domain and range of $g(x)$.
 - (b) Sketch the graph of $f(x) = -g(x - 1) + 1$ on the graph. Make sure the endpoints and corner point are in exactly the right locations.



9. Use algebraic methods to determine if the following functions are odd, even or neither. Show all your work.
 - (a) $f(x) = 4x + |x|$
 - (b) $f(x) = x^2 - |x| + 1$
 - (c) $f(x) = x^7 + x$

10. Use the graph to the right to answer the following questions.

- What is the domain of the function?
- What is the range of the function?
- Find an equation for $f(x)$.



11. Answer the following about the functions given below.

$$f(x) = \frac{1}{x+3}, g(x) = x^2 - 2$$

- Determine $f(g(3))$
- Determine the function $g(f(x))$
- Determine $f(f(1))$

12. The zombie apocalypse is upon us. In order to save you and your friends, you need to construct an enclosure to trap them. You manage to find 100 ft of fencing and want to build a rectangular enclosure next to a river. (Hopefully, zombies can't swim)

- Determine the area of the enclosure as a function of x .
- What is the maximum area you can enclose with the amount of fence you salvaged?

13. Kim's Cookie Emporium sells cookies online. When she prices the cookies at \$2.50 per box, she sells 150 boxes per day. For every \$0.25 reduction in price, she sells an additional 25 boxes per day.

- Find price as a function of x where x is the number of cookies sold per day.
- Find the revenue function $R(x)$ for Kim's business for a 30 day month. (Revenue is price times quantity)
- How many boxes per day should Kim sell to maximize revenue?
- The monthly cost function for Kim's business is given by $C(x) = 700 + 1.5x$ where x is the average number of boxes sold per day for the month. Find the profit function $P(x)$ for Kim's business. (Hint: Profit is revenue minus cost)
- How many boxes should Kim sell per day to maximize profit?

Answers to Practice Problems

1. Point $P(-2,3)$ and $Q(8,-5)$

- (a) $\sqrt{164}$
- (b) $(3,-1)$
- (c) $R(18,-13)$

2. $(x+2)^2 + (y+8)^2 = 16$

3. $C(3,-4), R = 6$

4. $f(x): [-7, \infty), g(x): (3, \infty)$

5. $m = 6, y = 6x - 17$

6. Binky's snow cones

- (a) $Sales = -80(Price) + 560$
- (b) 80 snow cones
- (c) \$1.75
- (d) Domain: $[0,7)$, Range: $[0,560)$

7. Point $P(1,2)$

- (a) New $P\left(\frac{1}{4}, -10\right)$
- (b) D: $[0, 5/2]$, R: $[-22, 0]$

8. Plot the graph

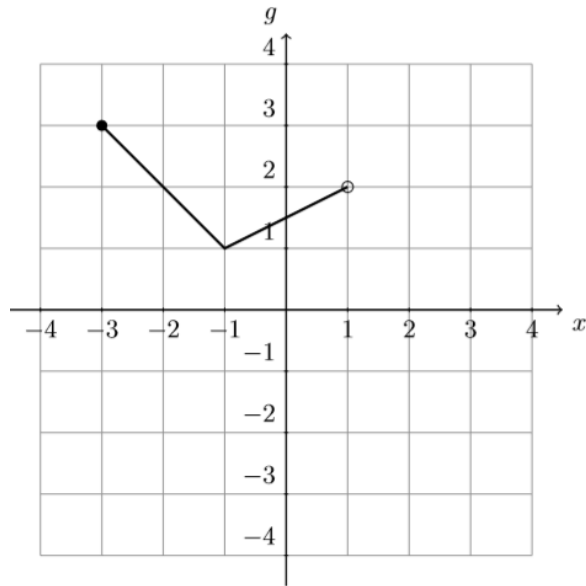
- (a) D: $[-3, 1)$, R: $[1, 3]$
- (b) See the graph to the right

9. Odd, even or neither?

- (a) Neither
- (b) Even
- (c) Odd

10. Piecewise graph

- (a) D: $[-4, 3]$
- (b) R: $[-1, 3]$
- (c)



$$f(x) = \begin{cases} x + 5, & -4 \leq x \leq -2 \\ 3x - 3, & -2 < x \leq -1 \\ -\sqrt{1 - x^2}, & -1 < x \leq 1 \\ -\frac{1}{2}x + \frac{1}{2}, & 1 < x \leq 3 \end{cases}$$

11. Compositions of $f(x)$ and $g(x)$

- (a) $\frac{1}{10}$
- (b) $g(f(x)) = \left(\frac{1}{x+3}\right)^2 - 2$
- (c) $f(f(1)) = \frac{4}{13}$

12. Zombies

- (a) $A(x) = 100x - 2x^2$
- (b) $1250 ft^2$

13. Kim's Cookie Emporium

- (a) $f(x) = -.01x + 4$
- (b) $R(x) = -.03x^2 + 120x$
- (c) 200 boxes
- (d) $P(x) = -.03x^2 + 118.5x - 700$
- (e) 197.5 boxes