Some helpful equations:

\[ A(t) = P(b)^t \quad A(t) = P(1 + r)^t \quad A(t) = P_{old}(P_{new}^{\frac{r}{100}})^{\frac{t}{n}} \quad A(t) = P(h)^{\frac{t}{L}} \]

- \( A \): amount after some time
- \( P \): “principle”, or the initial amount
- \( b \): growth/decay factor
- \( t \): time
- \( n \): number of times compounded per year
- \( L \): how long something took
- \( r \): growth/decay rate

1. The following table shows Makebelievia’s Vibranium production for the years between 2000 and 2005. Let \( t=0 \) represent the year 2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibranium</td>
<td>39</td>
<td>63</td>
<td>101</td>
<td>144</td>
<td>199</td>
<td>280</td>
</tr>
</tbody>
</table>

(a) Find the linear model that best fits this data.
(b) Find the exponential model that best fits this data. What is the initial value, what is the growth factor, what is the growth rate?
(c) Which of these models is a better choice, use it for the rest of these questions
(d) The model [overpredicts — underpredicts — exactly predicts] 2002’s vibranium production.
(e) What is the SSE, average error and correlation coefficient?
(f) When does the vibranium production exceed 1000?
(g) Predict the vibranium production in 2015.

2. A mathematician cooks a cake in a 350 degree oven. The table below shows the internal temperature of the cake.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>°Cake°F</td>
<td>67.7</td>
<td>114.8</td>
<td>154.1</td>
<td>186.6</td>
<td>214.9</td>
<td>236.1</td>
</tr>
</tbody>
</table>

(a) Find the Newton’s Law of Heating equation for this data.
(b) Find the SSE, average error, and correlation coefficient for this model.
(c) Predict the cake’s temperature when the mathematician pulls it out at 45 minutes. (Extra credit: Predict the look on the mathematician’s face when he pulls this “cake” out)
(d) What is the exact time this cake passes 212 degrees F.

3. Makebelievia Credit Union offers a savings account with 3.8% interest compounded monthly. Pietro opens an account with $2123.

(a) How much is the account worth in 6 months?
(b) When does the account reach $3000?
(c) What is the expected annual yield?

4. Makebelievia Credit Union also offers a Certificate of Deposit with 7.3% interest compounded continuously. Wanda opens an account with $6382.

(a) How much is the account worth in 2 years?
(b) When does the account reach $10,000?
(c) What is the expected annual yield?

5. Which of these accounts has the best return on investment?

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>8.72%</th>
<th>8.32%</th>
<th>8.07%</th>
<th>7.94%</th>
<th>7.61%</th>
<th>7.50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compounded</td>
<td>annually</td>
<td>quarterly</td>
<td>monthly</td>
<td>weekly</td>
<td>daily</td>
<td>continuously</td>
</tr>
</tbody>
</table>
6. Palladium-103 has a halflife of 17 days. The reactor that powers the capital of Makebelivia has 23.1 kg of Palladium on January 1, 2016.

(a) How much palladium is there on March 1st (60 days later)?
(b) When does the amount of palladium fall below 7 kg?
(c) What is the daily decay rate?
(d) What is the continuous daily decay rate?
(e) What is the monthly (30 days) decay rate?

7. Does the following data match a linear function, exponential function, logarithmic function, or none of these?

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>7.100</td>
<td>3.950</td>
<td>.800</td>
<td>-2.350</td>
<td>-5.500</td>
<td>-8.650</td>
</tr>
</tbody>
</table>

8. Does the following data match a linear function, exponential function, logarithmic function, or none of these?

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3.100</td>
<td>3.720</td>
<td>4.464</td>
<td>5.357</td>
<td>6.428</td>
<td>7.7137</td>
</tr>
</tbody>
</table>

9. Does the following data match a linear function, exponential function, logarithmic function, or none of these?

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2.460</td>
<td>3.257</td>
<td>3.723</td>
<td>4.054</td>
<td>4.311</td>
<td>4.521</td>
</tr>
</tbody>
</table>

10. Use this matrix for the following questions:

\[
\begin{bmatrix}
1 & 3 & 0 \\
4 & 2 & 2 \\
0 & -1 & 1
\end{bmatrix}
\]

(a) What is the dimension of this matrix?
(b) What is element \(a_{1,2}\)?
(c) Does this have an inverse? If so, what is element \(a_{1,3}\)?

11. Use this matrix for the following questions:

\[
\begin{bmatrix}
1 & 3 & 0 \\
4 & 2 & 2 \\
0 & -1 & 1 \\
7 & 3 & 1
\end{bmatrix}
\]

(a) What is the dimension of this matrix?
(b) What is element \(a_{4,1}\)?
(c) Does this have an inverse? If so, what is element \(a_{2,3}\)?

12. Use this matrix for the following questions:

\[
\begin{bmatrix}
1 & 3 & 2 \\
2 & 1 & -1 \\
0 & -1 & -1
\end{bmatrix}
\]

(a) What is the dimension of this matrix?
(b) What is element \(a_{1,2}\)?
(c) Does this have an inverse? If so, what is element \(a_{2,1}\)?
Answers:

1. (a) \( V(t) = 47.3143t + 19.3809 \)
   (b) \( V(t) = 42.273 \times 1.47746^t \), initial value = 42.273, growth factor = 1.47746, growth rate = 47.746%
   (c) Exponential \( (r^2 \text{ is closer to 1}) \)
   (d) underpredicts
   (e) SSE = 461.66, average error = 9.61, correlation coefficient = .9959
   (f) \( t = 8.11 \) so 2009
   (g) 14750.10

2. (a) \( D(t) = 282.12 \times .9641^t \)
   (b) Find the SSE = 1.2307, average error = .4961, correlation coefficient = -.9999
   (c) \( D = 54.65, T = 295.35 \). (Extra credit: :'( )
   (d) \( t = 19.61 \)

3. (a) \$2163.66
   (b) \( t = 9.11, 9 \text{ years and 3 months} \)
   (c) 3.867%

4. (a) \$7385.22
   (b) \( t = 6.15 \)
   (c) 7.573%

5. E.A.Y.'s:

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>8.72%</th>
<th>8.32%</th>
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<th>7.94%</th>
<th>7.61%</th>
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<td>weekly</td>
<td>daily</td>
<td>continuously</td>
</tr>
<tr>
<td>E.A.Y.</td>
<td>8.72%</td>
<td>8.58%</td>
<td>8.37%</td>
<td>8.26%</td>
<td>7.91%</td>
<td>7.79%</td>
</tr>
</tbody>
</table>

Annual is best.

6. (a) 2.0 kg
   (b) \( t = 29.28 \)
   (c) \( r = -3.995\% \)
   (d) \( r = 4.077\% \)
   (e) \( r = -79.544 \)

7. linear

8. exponential

9. logarithmic

10. (a) 3x3
    (b) 3
    (c) yes, -.75

11. (a) 4x3
    (b) 7
    (c) no

12. (a) 3x3
    (b) 3
    (c) no