Exam 2

By providing my signature below I acknowledge that this is my work, and I did not get any help from anyone else:

Name (sign): ___________________________ 
Name (print): ___________________________

Student Number: _______________________

Instructor’s Name: _______________________
Meeting Time: _________________________

• Please show your work. An unjustified answer may receive little or no credit.

• Your work must be neat. If I can’t read it (or can’t find it), I can’t grade it.

• The exam is worth a total of 100 points. The value of each problem is shown at the beginning of each problem.

• Please turn off your mobile phone.

• You are only allowed to use a TI-30 calculator. No other calculators are permitted.
1. Solve the following equations for $x$:

(a) (6 pts) $2^{x+1} = 8^x$

(b) (10 pts) $2^{5x}5^{x+1} = e^x$

(c) (10 pts) $2 \log_3(x - 2) - \log_3(4x) = 2$. 
2. (12 pts) Find the inverse, domain, and range of \( f(x) = \frac{2x+5}{7-3x} \).

3. (12 pts) Sketch the graph of \( f(x) = \log_2(x + 1) - 2 \) below. Mark two points explicitly, as well as its asymptote.
4. \( (15 \text{ pts}) \) You’ve got a fever, a fever for logarithms. Even though it’s incurable, the doc tried to give you some medicine to treat it. You took 300 milligrams of this totally ineffective medicine three hours ago, and only 100 milligrams of it are left in your body now. Assuming the decay in this medicine is exponential, how much longer will it be before there is only 10 milligrams of the medicine left?
5. You have two job options. ACME offers you a base pay of $35,000 per year, plus $10,000 in special company stock whose value grows at a continuously compounded rate of 8%. Turtle Wax Inc. offers you $40,000 per year, but no stock.

(a) \((4 \text{ pts})\) Let \(T\) be cumulative value of your compensation \(t\) years after you start working for Turtle Wax Inc. To be clear, the value of \(T\) should be cumulative, so for example when \(t = 1\) you should get \(T = 40,000\) and when \(t = 2\), you should get \(T = 80,000\). Find an equation which expresses \(T\) as a function of \(t\).

(b) \((10 \text{ pts})\) Let \(A\) be the cumulative value of your total compensation (salary paid+stock value) at ACME \(t\) years after you were hired. In other words, \(A\) is the total cash you’ve earned in salary + the value of your stock at time \(t\). Find an equation which expresses \(A\) as a function of \(t\).

(c) \((6 \text{ pts})\) Assuming you earn no raises from either company, which is the better to work for if you only plan to stay for one year? What about two years? What about ten years?
6. (15 pts) For each of the following “real life” situations, state if it is best modeled by an exponential, logarithmic, linear, or quadratic function. Don’t say “none” and then come up with philosophical excuses. Just tell me the best one and say something to justify it if you want a shot at partial credit.

(a) You come to a casino with $500 and play black jack. You bet $10 on every hand and lose every time until the money’s gone. What kind of function best describes your money as a function of time?

(b) You come to a casino with $500 and play black jack. You bet half of the money you have left on every hand and lose every time until the money’s gone. What kind of function best describes your money as a function of time?

(c) You want to build a circular enclosure, and fencing costs $2 per linear foot. What kind of function best describes the area of the enclosure as a function of how much money you spend on fencing?

(d) You are staging an athletic tournament. Every round you pair off all the teams, and the losing teams are eliminated. Let \( x \) be the number of teams in your tournament, and let \( y \) be the number of rounds that need to be played before the championship game. What kind of function best describes \( y \) as a function of \( x \)?

(e) Your faucet has been left on and is pouring water into your parabolic shaped sink. What kind of function best describes the total volume of water in your sink as a function of time?