Things you know:

$\sum_{n=0}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$

$\sum_{n=0}^{\infty} ar^n$ converges if $|r|<1$

diverges if $|r|\geq 1$

$\sum_{n=0}^{\infty} \frac{1}{n^p}$ converges if $p>1$

diverges if $p\leq 1$

$\sum_{n=0}^{\infty} (-1)^n a_n$ converges if

$\lim_{n\to\infty} a_n = 0$ and $a_n > a_{n+1}$

$\sum_{n=0}^{\infty} a_n$ converges if

$\int_0^{\infty} a_n < \infty$

Start with a problem

$\sum_{n=0}^{\infty} a_n = ?$

nth-term test

$\lim_{n\to\infty} a_n$

$\lim_{x\to\infty} a_n \neq 0$ → Diverges

$\lim_{x\to\infty} a_n = 0$

Does it have n in the exponent or !'s?

yes → Ratio or root test $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

succeeds (< 1) → Converges absolutely

fails (>1)

inconclusive(=1)

no

Does it have polynomials or logs?

yes → Compare to p-series $a_n \leq b_n$ and $\sum b_n$ converges

succeeds → Converges absolutely

fails

Inconclusive

no

Integral test

$\int a_n = b$ → Converges absolutely

Doesn't work

Alternating Series Test

succeeds → Converges conditionally

fails → Diverges

Common Tests:

Ratio Test:

$\sum_{n=0}^{\infty} a_n$ converges if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

and diverges if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$

Root Test:

$\sum_{n=0}^{\infty} a_n$ converges if $\lim_{n\to\infty} \sqrt[n]{a_n} < 1$

and diverges if $\lim_{n\to\infty} \sqrt[n]{a_n} > 1$